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Gauge Theory of Antisymmetric Tensor Fields

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Abstract

The compact lattice gauge theory of the second-rank antisymmetric tensor field in four dimensions is studied. By Monte-Carlo simulation on a 10^4 lattice, we calculate the hypersurface tension. It is found that the tension does not go to zero for a finite value of the coupling constant. This suggests the theory has only the confinement phase. This result is consistent with the result obtained from the dilute gas approximation of the instanton.

Some phenomenological implications on the string model are also discussed.

1. Introduction

It is well known that the compactifications on the Calabi-Yau manifolds in superstring theories lead to an enormous number of four dimensional models which possess $N = 1$ space-time supersymmetry. Unfortunately, due to the lack of the understanding in the non-perturbative aspects of the string theory we have not yet succeeded in making low energy predictions from those models. So far one is only able to treat low energy effective field theories to do some phenomenological analyses. Although such analyses might give new predictions for a certain class of models, they would not clarify the novel model-independent features which are low energy remnants peculiar to string theory.

On the other hand one of the model independent features in string theory is the appearance of the second rank antisymmetric tensor field which is expected to behave as an almost massless scalar boson at low energy. Moreover owing to the anomaly cancelling counterterm this scalar boson has the same low energy Lagrangian as that of the invisible axion. Therefore it would be of extreme importance to study what the fate of this would-be invisible axion is.

The cosmological observations impose severe constraints on the value of the coupling constants of the invisible axion to matter fields. The coupling constant has a dimension of $(mass)^{-1}$, where the scale of this mass is restricted to be $10^9 \sim 10^{12}$ GeV[1,2,3]. The tree-level mass scale of the coupling of the superstring axion is the Planck scale. If the tree-level Lagrangian for the superstring axion remains to be correct down to the low energy scale, there will be a serious contradiction to the cosmological constraints.

Since the model independent axion comes from the second-rank antisymmetric tensor gauge field, the gauge invariance guarantees the axion to be massless at least in the perturbative sense. One would then imagine that some non-perturbative effect may change the low energy behavior of the antisymmetric tensor field drastically. In fact this possibility is the point we consider in this paper. In 1982, Peter Orland[4] studied compact antisymmetric tensor gauge theory using the instanton

approximation and found that the system is in a confinement phase for any value of the coupling constant. To go beyond the instanton approximation, one has to study the model in the lattice approach.

In this paper we study the compact lattice model by calculating the expectation value of the Wilson surface. In Section two we formulate the lattice theory. In Section three results of the numerical simulations are presented. Section four is devoted to the physical implications of the results to string theory. Conclusions and discussions are given in Section five.

2. Compact Lattice Gauge Theory of Antisymmetric Tensor Field

We formulate the lattice gauge theory of the antisymmetric tensor field. The partition function is given by the following path integral.

$$Z = \int \prod_p d\theta(p) exp(-S_g) \tag{1}$$

where

$$S_g = \sum_c \beta(1 - cos(\sum_{p \in c} \theta(p))) \tag{2}$$

Here c and p denote unit cubes, plaquettes. The lattice action has the invariance under the following gauge transformation

$$\theta_{\mu\nu}(i) \longrightarrow \theta_{\mu\nu}(i) + \Lambda_\nu(i+\mu) - \Lambda_\nu(i) - \Lambda_\mu(i+\nu) + \Lambda_\mu(i). \tag{3}$$

If we take a naive continuum limit by taking $\theta_{\mu\nu}(i) = a^2 B_{\mu\nu}(x)$, where a is the lattice spacing, the action takes the following form in the continuum,

$$S = 3\beta a^2 \int d^4x H_{\mu\nu\lambda} H^{\mu\nu\lambda}. \tag{4}$$

The tensor $H_{\mu\nu\lambda}$ is the gauge invariant field strength defined as $H_{\mu\nu\lambda} = \partial_{[\mu} B_{\nu\lambda]}$ which is invariant under $\delta B_{\mu\nu} = \partial_{[\mu} \Lambda_{\nu]}$.

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The gauge invariant observables are the Wilson surfaces given below

$$W(S) = \prod_{p \in S} exp(i\theta(p)).$$ (5)

We can calculate the vacuum expectation value of the Wilson surface analytically in the strong coupling limit and the weak coupling limit

$$\langle W(S) \rangle \sim_{\beta \to 0} (\frac{\beta}{2})^V$$

and (6)

$$\sim_{\beta \to \infty} exp(-\frac{1}{6\beta a^2} \int_S d\Sigma^{\mu\nu}(x) \int_S d\Sigma^{\rho\sigma}(x') \langle B_{\mu\nu}(x) B_{\rho\sigma}(x') \rangle)$$

respectively. Here V is the minimum volume surrounded by the surface S.

When the surface S is that of the unit cube c, the vacuum expectation value of W(S) ($= \langle W(c) \rangle$) is related to the vacuum expectation value of the energy per unit cube in the following way

$$\langle E \rangle = 1 - \langle W(c) \rangle.$$ (7)

Now since in the strong and the weak coupling limit the partition function behaves as,

$$Z \sim_{\beta \to} e^{-\beta N_c}(1 + \frac{\beta^2}{4} N_c),$$

and (8)

$$\sim_{\beta \to \infty} \beta^{-\frac{1}{2}(N_p - N_l - N_s)},$$

respectively, the energy per unit cube is

$$\langle E \rangle = -\frac{1}{N_c} \frac{\partial}{\partial \beta} ln Z$$

$$\sim_{\beta \to 0} 1 - \frac{\beta}{2}$$ (9)

$$\sim_{\beta \to \infty} \frac{3}{8\beta}.$$

Here N_c, N_p, N_l and N_s are the number of cubes, plaquettes, links and sites respectively. We have used the fact that for N^4 lattice, $N_c = 4N^4, N_p = 6N^4, N_l = 4N^4$

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and $N_s = N^4$.

Another important physical observable is the correlation function of the field strength

$$\langle H_{\mu\nu\lambda}(x)H_{\rho\sigma\kappa}(x')\rangle. \tag{10}$$

In the lattice theory it can be obtained by calculating the correlation function of the small Wilson surfaces:

$$\begin{aligned}
\langle W(S)W(S')\rangle &- \langle W(S)\rangle \cdot \langle W(S')\rangle \\
&= \langle W(S)\rangle \cdot \langle W(S')\rangle (exp(-\langle H_{\mu\nu\lambda}(x)H_{\rho\sigma\kappa}(x')\rangle) - 1) \\
&\sim \langle H_{\mu\nu\lambda}(x)H_{\rho\sigma\kappa}(x')\rangle.
\end{aligned} \tag{11}$$

where S and S' are the surfaces of unit cubes located at the points x and x'. In the strong coupling limit the correlation function behaves as

$$\langle H_{\mu\nu\lambda}(x)H_{\rho\sigma\kappa}(x')\rangle \sim \beta^{6|x-x'|} \tag{12}$$

which suggests a massive particle mediates inter-string interactions.

The intuitive picture of the result obatined in the strong coupling expansion is that the antisymmetric tnesor field works as an intra-string binding force which is proportional to the area of the region surrounded by the string. This is the "volume law " of the Wilson surface. On the other hand, the inter-string interaction is suppressed by this "confinement effect", and the only possible interaction is the short-range van der Waals force of the antisymmetric tensor field. This picuture is consitent with the result obtained by instanton approximation[4]which we introduce in Section 4.

In order to see whether this picture remains true for any value of β, numerical

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study is required, which will be presented in the next section.[*]

3. Simulations

In this section we calculate the vacuum expectation values of the energy density and the Wilson surface for the compact lattice gauge theory of the second rank antisymmetric tensor gauge field by Monte-Carlo simulations. The number of lattice sites is 10^4. Our algorithm is the Metropolis algorithm. Using KEK supercomputor HITAC S820, we have performed 10,000 sweeps for each value of the coupling constant.

In Fig. 1, the Wilson surface per unit cube $\langle W(c) \rangle (= 1 - \langle E \rangle)$ as a function of β is presented. We see that there is no sign of any phase transition. The result agrees with that of the strong coupling limit and the weak coupling limit for sufficiently small and large β respecetively. Pearson[5]caluläted the thermal loop of $ln(\langle exp(\beta \prod_{p \in c} e^{i\theta(p)}) \rangle)/\beta$ and found qualitaively the similar result.

Next we considered $I \times J \times K$ rectangulalr solids (I,J,K are integers) for which the vacuum expectaion values of the Wilson surfaces $W_{I,J,K}$ are calculated. For large I,J and K, $W_{I,J,K}$ in general behave as

$$W_{I,J,K} \sim exp(-\chi \cdot IJK - a \cdot (IJ + JK + KI) - b \cdot (I + J + K)), \qquad (13)$$

where χ is the hypersurface tension and a, b are some constants. The hypersurface tension is obtained from the generalization of the Creutz ratio as follows

$$\chi(I, J, K) = -ln(\frac{\langle W_{I,J,K} \rangle \langle W_{I,J-1,K-1} \rangle \langle W_{I-1,J,K-1} \rangle \langle W_{I-1,J-1,K} \rangle}{\langle W_{I-1,J-1,K-1} \rangle \langle W_{I-1,J,K} \rangle \langle W_{I,J-1,K} \rangle \langle W_{I,J,K-1} \rangle}). \qquad (14)$$

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[*] The antisymmetric tensor Z_N lattice gauge theories are equivalent to spin systems. In four dimensions they have phase transitions at finite β. On the other hand, the antisymmetric tensor U(1) lattice gauge theory is equivalent to a coulomb gas system of magnetic monopoles[6,7]. The phase structure could be different in theories with Z_N symmetry and U(1) symmetry. This is actually the case for Z_N and U(1) gauge theories in three dimensions. As N gets larger the phase transition point *beta* in Z_N lattice gauge theories gets larger, and in the limit $N \rightarrow \infty$ (namely $Z_\infty \equiv$ U(1) gauge theory) the phase transition point blows up[8]so that there is no phase transition in the limiting case.

We have attached the indices I, J, K to the hypersurface tension to remind us for which size of the surface the hypersurface tension is calculated. We plotted the hypersurface tensions $\chi(1,1,1), \chi(2,2,2), \chi(3,3,3)$ and $\chi(4,4,4)$ for various values of β in Fig.2. The hypersurface tension does not seem to go to zero for a finite value of β. Due to low statistics we could not determine the scaling behavior of the hypersurface tension. We will make a comment on the possible continuum limit in the next section. In any way the above result is a strong evidence that the system is in the confinement phase for any value of the coupling constant.

We could not obtain the mass which appears in the field strength correlation function because of low statistics.

4. Physical Implications to String Theory in Four Dimensions

In the last section we have presented the result of the Monte-Carlo simulation of a lattice gauge theory of the antisymmetric tensor field. How can we interpret the result ? Since the naive continuum limit of the lattice theory presented in the last section is a trivial free field theory, one might think that the dynamical structure like the confinement cannot exist. However, this statement is not necessarily true. In fact the topological excitations can play a crucial role.

Suggested by the work of Polyakov[3] in the compact QED in three dimensions, Peter Orland[4] studied n-th rank antisymmetric tensor gauge theories in n+2 and other dimensions. The case with n=1 is the compact QED in three dimensions and the case with n=2 is the second rank antisymmetric tensor gauge theory which we are studying in this paper. Assuming that the dilute gas approximation for instanton is legitimate, he calculated the vacuum expectation values of the Wilson surface $\langle W(S) \rangle$ and the correlation function of the field strength $\langle F_\mu F_\nu \rangle$ for the case with n=2, where $F_\mu \equiv \epsilon_{\mu\nu\lambda\kappa} H^{\nu\lambda\kappa}$. Introducing a cutoff scale a^{-1}, they are expressed as follows,

$$\langle W(S) \rangle \sim exp(-\mu(g)A),$$
$$\langle F_\mu F_\nu \rangle \sim \frac{k_\mu k_\nu}{k^2 + M^2}, \tag{15}$$

where $\mu(g)$ is the hypersurface tension and M is the mass of the scalar mode which are given by the following equations

$$\mu(g) = a^{-3}\beta^{\frac{-1}{2}}e^{-\frac{1}{2}\beta S_{inst}},$$
$$M = a^{-1}\beta^{\frac{1}{2}}e^{-\frac{1}{2}\beta S_{inst}}. \tag{16}$$

If we take the continuum limit $a \rightarrow 0$ for finite βa^2, the hypersurface tension and the mass vanish and we obtain a trivial theory. On the other hand, if we fine tune the coupling constant in such a way that the hypersurface tension or the mass remains finite, we obtain a non-trivial theory. In that case the continuum action turns out to be the sine-Gordon action.

Now let us make a rough estimation in the four dimensional string model. Although the actual string model is more complicated than the pure $B_{\mu\nu}$ gauge theory, we think our model describes one of the essential features of the low energy behavior of the antisymmetric tensor field in the string theory. Assuming eq.(16)be correct, we now estimate the scale of the hypersurface tension and the mass of the scalar mode by substituting the coupling and cutoff scale. What is the coupling constant and the cutoff scale in the string theory? The standard compactifcation scenario on Calabi-Yau manifold, when the string is weakly coupled, gives[10]

$$\beta = 1/g^2 = 1/(4\pi\alpha GUT) \sim 10$$
$$a = 1/M_{compactification} \sim (10^{15} - 10^{19} GeV)^{-1}. \tag{17}$$

The action for the instanton S_{inst} may vary drastically depending on the details of the regularization. If we take $\frac{1}{2}S_{inst} = 1$,

$$\mu(g) \sim (10^{13} - 10^{17} GeV)^3,$$
$$M \sim 10^{10} - 10^{14} GeV. \tag{18}$$

The mass scale becomes even smaller if S_{inst} is larger. In any case we obtain a string confinement at scales much below the Planck scale and the massive scalar whose mass is much smaller than the Planck scale. The physical result of the confinement effect by the antisymmetric tensor in string theory would be the following :

1. According to the field strength correlation function there exist only massive modes in the antisymmetric tensor field theory. It is expected that this state could decay into many states, because its mass is quite large compared to the QCD scale. Therefore the antisymmetric tensor field, which was first expected to generate a weakly interacting axion that contradicts cosmological observations, is now harmless.

2. It is expected that the binding energy of the string due to the hypersurface tension may change the low lying particle spectrum of the string theory. This may be one of the mechanism of obtaining a small mass scale in string theory. However one needs more elaborate study to obtain the actual modifications of the particle spectrum.

Now three remarks are in order. In the above estimation we have neglected the existence of the string excited states whose mass is of the Planck scale or the compactification scale. However, as long as we are interested in the low energy behavior of the antisymmetric tensor field, the quantum effect of those higher modes are irrelevant. In that respect our analysis may be viewed as a Quenched approximation of the full theory.

Secondly, we have also neglected other massless fields like the graviton, the dilaton and the gauge bosons. It would be very important to study the effects of the gravity and the dilaton in the lattice approach, but this will be left as a future work.

In the present work we chose a compact lattice action instead of non-compact lattice action. This is simply because it is obvious that in the compact lattice theory the topological excitations such as the instantons, which plays a crucial role in the confinement, are allowed.

5. Discussions

In this paper we studied the compact lattice gauge theory of the antisymmetric tensor gauge field in four dimensions. By calculating the energy density and the hypersurface tension, we confirmed that the theory is in a confinement phase for

any values of the coupling constant. This result is consistent with the earlier lattice calculation and give qualitaively the same picture as those obained by the instanton analysis. We have also discussed some phenomenological implications to string models.

Much remain to be done in this topic. Estimating the corrections to particle spectrum below the confinement scale is important. This will be studied in the future.

The effect of other massless fields also has to be studied. Some interesting results along this line were obtained by S.J.Rey[11]who found an instanton solution of antisymmetric tensor field coupled to gravity and dilaton field. While typing this manuscript the authors received a preprint[12]in which instanton solutions involving the antisymmetric tensor field and a scalar field is studied.

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Figure Captions

Figure 1

The expectation values of the Wilson surface for a unit cube $\langle W(c)\rangle (= 1 - \langle E \rangle)$ as a function of β are presented. The numerical result agrees with the analytical behavior obtained in the strong and the weak coupling limit.

Figure 2

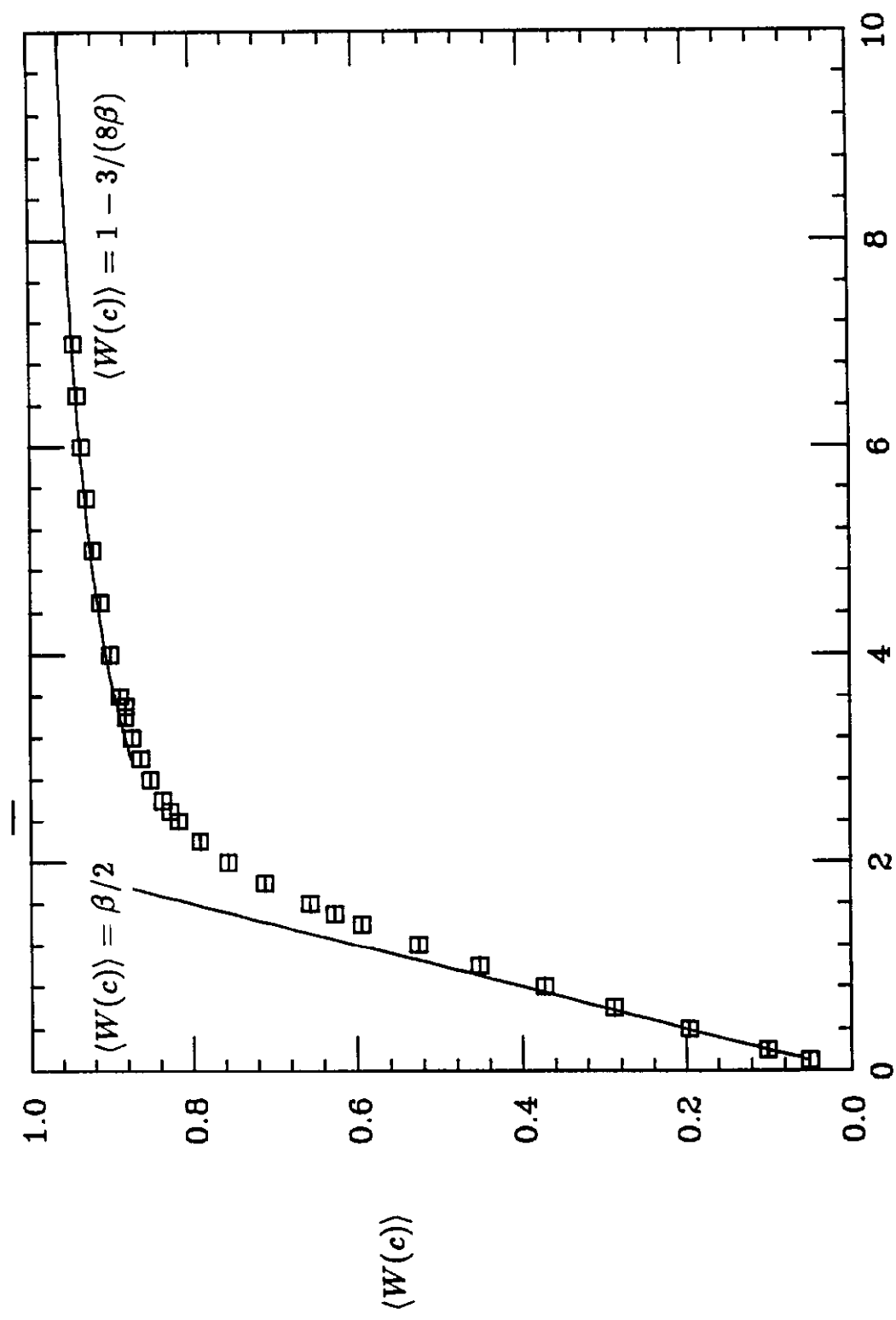The hypersurface tension for various values of β is plotted. The tension takes a non zero value for any β.
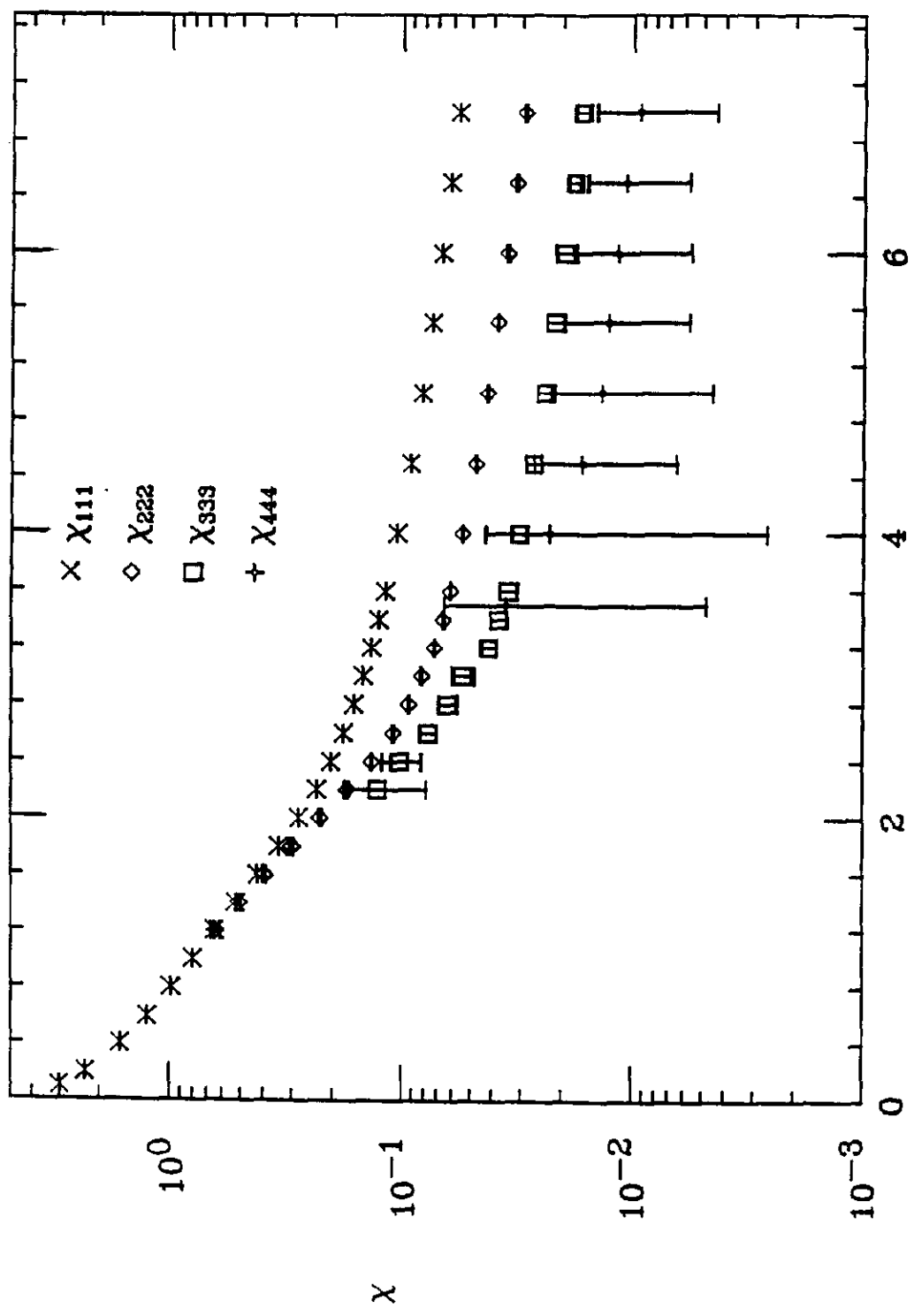
$\langle W(c) \rangle = 1 - 3/(8\beta)$

$\langle W(c) \rangle = \beta/2$

Fig.1

Fig.2